ABSORPTION OF MONOCHROMATIC RADIATION BY A GAS INJECTED IN THE VICINITY OF THE STAGNATION POINT OF A BODY IN SUPERSONIC FLIGHT

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The authors theoretically investigate the absorption of radiation in a blown layer.

References [1-6] investigated the screening of surfaces of bodies washed by a gas flow from external radiation with the aid of blowing of a foreign gas from their surfaces. References [1-4] studied flow over a flat plate at zero angle of attack of near supersonic gas, with low-level blowing of gaseous SF_6 into the boundary layer (radiation at λ = 10.6 μm falls normally on the plate). Reference [4] presented results of solving a model problem associated with hypersonic flow over a wedge. Reference [6] investigated the interaction of monochromatic radiation with the layer of absorbing gas blown from the surface of a blunted body.

The present paper considers screening of the surface of a blunt body from monochromatic radiation of λ = 10.6 μm with the aid of strong blowing of SF_6. We seek a solution in the vicinity of the stagnation stream line about an axisymmetric body where, as shown in [6], the radiative flux distribution at the wall has a maximum. We give the appropriate equations, describe the numerical solution, show the results, and propose an approximate method of solving this problem.

1. We consider axisymmetric flow about a blunt body of revolution washed by hypersonic flow of a gas, with strong blowing of a foreign gas from its surface [6-8]. On the shock wave surface falls monochromatic radiation parallel to the body symmetry axis of such a wavelength that the shock layer is optically transparent and, conversely, the blown gas is a good absorber.

The thermal radiation from the hot part of the shock layer is small compared with the monochromatic radiation. In the mathematical description of the flow the boundary layer between the blown gas and the external flow is replaced by a contact surface. The flow in the blown layer is equilibrium and inviscid. We consider that the gas temperature increase in the blown layer due to radiation absorption is moderate, and therefore all the flow parameters [6] are of the same order as in the absence of radiation [6-9].

We investigate the flow region near the stagnation line $x^2/\omega \ll 1$ (x = 0 corresponds to the body stagnation point). From the fact that the radiation is parallel to $\mathbf{V}_\infty$, that there is no reradiation or reflection from the wall, and bearing in mind the considerations presented in [6] and the fact that both terms of the continuity equation are of the same order, we can write a system of equations for the flow in the blown layer:

$$\frac{\partial}{\partial x}(\rho u r^{\nu-1}) + \frac{\partial}{\partial y}(\rho v r^{\nu-1}) = 0,$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x}, \qquad \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y},$$

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = -\frac{1}{r^{\nu-1}} \frac{\partial}{\partial y}(q_y r^{\nu-1}), \tag{1}$$

$$\frac{\partial I}{\partial y} = -kI, \quad p = \rho R_g T, \quad q_y = I \cos\alpha(x), \quad h = \int_0^T c_p(\lambda)\, d\lambda.$$

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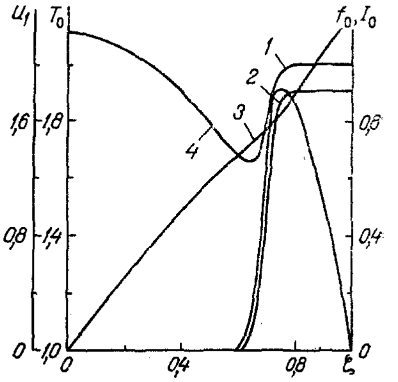

Fig. 1

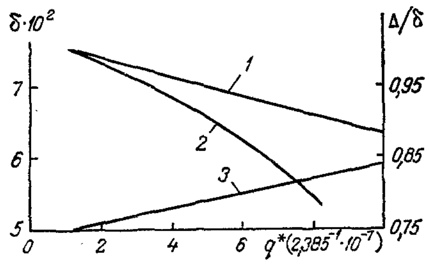

Fig. 2

Fig. 1. Distribution of the parameters in the blown layer for $|q_\infty| = 1.908 \cdot 10^8$ W/m², $\rho_\infty = 0.74$ kg/m³, $V_\infty = 2 \cdot 10^3$ m/sec, W = 1, $R_c = 0.1$ m, $T_\infty = 300$°K: 1) $I_0(\zeta)$; 2) $T_0(\zeta)$; 3) $u_1(\zeta)$; 4) $f_0(\zeta)$.

Fig. 2. Standoff distance of the contact surface and location of the absorption zone: 1) δ (numerical computation); 2) Δ/δ (numerical computation); 3) δ (approximate method). The quantity $[q^*] = \mathrm{W/m^2}$.

TABLE 1. Influence of T_w and W on the Radiant Flux at the Body Surface ($\rho_\infty = 0.74$ kg/m³, $V_\infty = 2 \cdot 10^3$ m/sec, $R_c = 0.1$ m, $|q_\infty| = 1.908 \cdot 10^9$ W/m²)

$\bar{q} = q_w/\lvert q_\infty\rvert$	0,9	0,8	0,7	0,6
W, $T_w = 300$ K	0,5	1,0	1,45	2,0
$T_w/300$ K, $W = 1$	1,3	1,0	0,83	0 75

Taking into account that $p \approx \rho_\infty V_\infty^2 (1 - \bar{x}^2 + \ldots)$, $\bar{x} = x/R_c$, setting the coordinate y = 0 to be the corresponding body surface, and following reasoning analogous to that in [10], we seek a solution of the system (1) in the form

$$\rho = \frac{\rho_\infty}{\omega^2}(\rho_0(\zeta) + \tilde{R}(\bar{x})\rho_2(\zeta) + \ldots),\quad v = \omega^2 V_\infty (f_0(\zeta) + V(\bar{x}) f_2(\zeta) + \ldots),$$
$$r = R_c(\bar{x} + \ldots),\quad I = -I^*(I_0 + \ldots),\quad h = h^*(h_0 + \ldots), \tag{2}$$
$$u = \omega V_\infty \bar{x}(u_1(\zeta) + \ldots),\quad \zeta = y/\delta R_c,\quad p = \rho_\infty V_\infty^2 (p_0(\zeta) - \bar{x}^2 p_2(\zeta) + \ldots).$$

Here δR_c is the displacement thickness at the stagnation line. Since the form of the solution of Eq. (2) was chosen in accordance with the boundary conditions on the body $\rho v|_w = \rho_w(x)V_w(x)$, $T|_w = T_w(x)$, $u|_w = 0$ and at the contact surface $I|_c = I_c(x)$, $p|_c = p_c(\bar{x})$, then, taking into account that the distribution of $T_w(x)$ and $\rho_w(x)V_w(x)$ is such that $R(\bar{x}) \ll \bar{x}^2$ and $V(\bar{x}) \ll \bar{x}^2$, substituting Eq. (2) into Eq. (1) and equating terms with the same powers of $\bar{x}$, we obtain:

$$-\frac{\nu\delta}{\omega}\rho_0(\zeta)u_1(\zeta) = [\rho_0(\zeta)f_0(\zeta)]',\quad \omega^2 = \frac{T_w^* R_g}{V_\infty^2},$$
$$p_0' = -\omega^2\rho_0(\zeta)f_0(\zeta)f_0'(\zeta),\quad p_0 = T_0\rho_0,$$
$$\rho_0(\zeta)u_1^2(\zeta) + \frac{\omega}{\delta}\rho_0(\zeta)f_0(\zeta)u_1'(\zeta) = 2\beta, \tag{3}$$
$$p_2 = \beta,\quad I_0' = \tau_0 k(p_0, T_0)I_0,\quad \tau_0 = k^*\delta R_c,$$
$$\frac{2h^*}{V_\infty^2\Gamma}\rho_0(\zeta)f_0(\zeta)h'(\zeta) = I_0'(\zeta),\quad \Gamma = \frac{2q^*}{\rho_\infty V_\infty^3}.$$

The spreading out of the blown gas occurs near the contact surface, and therefore the constant describing the pressure drop along the body surface may be taken equal to $\beta = p_2(1)$. In turn $p_2(1)$ is equal to the analogous quantity on the other side of the contact surface [10]: $\beta = (\nu + 2)/(\nu + 1)$.

The boundary conditions convert to the following:

$$\zeta=0,\ u_1=0,\ T_0=1,\ \rho_0 f_0=W;\ \zeta=1,\ p_0=1-\frac{\varepsilon}{2},\ \varepsilon=\rho_\infty/\rho_s,\ f_0=0,\ I_0=1. \tag{4}$$

We note that for the five differential equations of the system (2) we have the six conditions (4), so that we can determine the unknown thickness of the blown layer.

2. In the radiation absent case the flow into the blown layer is described by the system of equations (3) in which we must replace the equations of radiative transfer and of energy by the conditions $I_0 = 0$, $T_0 = 1$. In addition, in Eq. (4) we omit the corresponding boundary conditions. Estimates show that for $M^2 \ll 1$ (M is the Mach number) we may assume, as was done in [7, 8, 10], that ρ = const. Then for $\nu = 2$, taking into account that $f_0(1) = 0$, we write

$$f_0=\frac{c_1}{4}(\zeta-c_2)^2-\left(\frac{2\delta}{\omega}\right)^2 A^2\frac{1}{c_1},\quad c_1=\frac{2}{c_2-1}\,\frac{2\delta}{\omega}A,$$
$$p_0=1-\frac{\varepsilon}{2}-\frac{\omega^2\rho_0}{2}f_0^2,\quad u_1=A\frac{\zeta-c_2}{1-c_2},\quad A=\left(\frac{2\beta}{\rho_0}\right)^{1/2}. \tag{5}$$

In determining δ and c_2 we obtain two solutions. One of these has no physical meaning ($\delta \to \infty$, $u_1(0) \to A$), and the other is expressed in the form

$$c_2=\frac{u_1(0)}{u_1(0)-A},\qquad \delta=\frac{\omega f_0(0)}{A+u_1(0)}. \tag{6}$$

Here we used the boundary conditions $u_1|_{\zeta=0} = u_1(0)$, $f_0|_{\zeta=0} = W/\rho_0$. We note that the solution of Eqs. (5) and (6) coincides with the results of [7] for $u_1(0) = 0$ and of [8] for $u_1(0) > 0$.

3. To calculate the influence of radiation on the flow in the blown layer we must numerically solve the system of equations (3), which is complicated by the presence of conditions (4) at opposite ends of the interval $\zeta = [0, 1]$. In this paper we use $\zeta = 0$ at one end of the interval of integration, besides the boundary conditions assigned there $u_1 = 0$, $T_0 = 1$, $\rho_0 f_0 = W$, the approximate values $p_0(0)$, $I_0(0)$, and also δ with subsequent improvement by iterations. Because of the strong influence of the quantities $p_0(0)$, $I_0(0)$, δ on the solution we had to arrange three iterations of the process, embedded one within the other, in which we first determined δ, then p_0 and finally I_0. We should emphasize that without embedding the iteration processes there was no convergence.

4. As was shown in [6], gaseous SF_6 satisfies all the conditions used in Sec. 1 in deriving the equations. In addition, the absorption coefficient has the form [1]:

$$k=k^*\bar{k},\quad k^*=\rho_\infty V_\infty^2 T^* C^*,\quad C^*=10^{-5}\ \mathrm{sec}^2/(\mathrm{kg\cdot K}),$$
$$\bar{k}=a_1\bar{p}(b-\bar{T}),\quad a_1=0{,}9653,\quad b=5{,}64\cdot 10^2/0{,}985\,T^*.$$

For $\bar{T} > b$ the absorption is curtailed and the gas becomes transparent for radiation of wavelength $\lambda = 10.6$ µm, and this means that the blown layer can be divided into two sublayers, a contact surface (I), where $\bar{T} = b$ and there is no absorption, and a sublayer (II), where the radiation is absorbed.

We can solve the problem, as described in Sec. 3, without dividing the blown region into two sublayers (as is done also with weak radiation, when the transparent region I is practically absent). However, we can accelerate the solution and improve the convergence if we carry out the numerical solution only up to $\bar{T} = b$, and then use the analytical solution given in Sec. 2. Here as boundary conditions for the analytical solution we take the values of the quantities $u_1(\Delta)$, $f_0(\Delta)$, $T_0(\Delta) = b$, where Δ is determined from the condition $T_0(\Delta) = b$ during the solution process.

The results of the numerical calculation are presented in Fig. 1. The increase of $f_0(\zeta)$ in the absorption zone is reminiscent of acceleration of a gas in a flame front. It can be

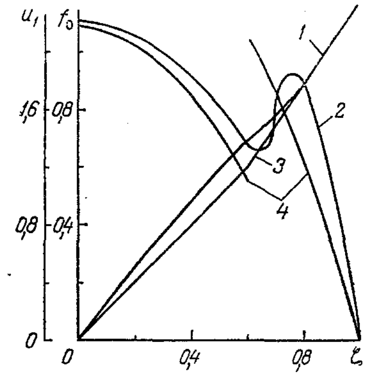

Fig. 3

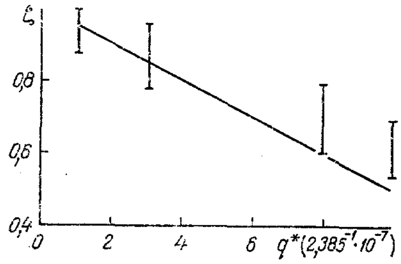

Fig. 4

Fig. 3. Distribution of the parameters in the blown layer, obtained numerically [1) $u_1(\zeta)$; 2) $f_0(\zeta)$] and by the approximate method [3) $u_1(\zeta)$; 4) $f_0(\zeta)$].

Fig. 4. Position and thickness of the absorption zone (intercept).

seen that the radiation is absorbed in a relatively narrow zone. The solutions on the left and right of this absorption zone behave analogously to the solutions of Sec. 2. Upon the increase of $|q_\infty|$ for some time $I_0(0) \approx 0$, after which it begins to increase sharply. Because of this the solutions presented in [6] can be used only for estimates. The calculations performed confirm the validity of the estimates of [6].

By varying $|q_\infty|$ with the other parameters held constant (see Fig. 1), we obtain $\delta(|q_\infty|)$ and $\Delta/\delta(|q_\infty|)$ (Fig. 2).

By varying T_w and W for the above parameters we obtain the result that q_w increases with increase of T_w and decreases with increase of W. By lowering the temperature of the blown gas we can substantially affect the screening process (see Table 1).

5. Analysis of a series of calculations (see, e.g., Fig. 1) indicates a relatively small thickness δ_{abs} compared with 1 of the part of the blown layer in which the temperature and the radiative flux vary respectively from 1 to b and from 0 to 1. Here in part of the blown layer we have $T_0 = b$, $I_0 = 1$ and the radiation is not absorbed, and in part $T_0 = 1$, $I_0 = 0$.

We shall find an approximate solution of the problem investigated, taking into account that $\delta_{abs} = 0$, and considering the blown layer to consist of two sublayers.

At the absorption front the energy release rate is q^*. We assume that the pressure is constant across the blown layer, $p^I = p^{II} = p_\infty V_\infty^2(1 - \varepsilon/2)$.

The balance relations at the absorption front have the form

$$\rho^I V^I = \rho^{II} V^{II}, \quad p^I = p^{II}, \quad \frac{V^{I^2}}{2} + h^I + \frac{q^*}{\rho^I V^I} = \frac{V^{II^2}}{2} + h^{II}, \tag{7}$$

where V^I, V^{II} are the gas velocities normal to the absorption front. The system of equations (7) reduces to an equation for m:

$$m^3 + dm + e = 0, \quad m = \rho^I V^I = \rho^{II} V^{II},$$
$$d = \frac{C_{p\,\mathrm{ef}}\, p^2}{R_g^2 (T^I + T^{II})}, \quad e = \frac{q^* p^2}{R_g^2 (T^{I^2} - T^{II^2})}. \tag{8}$$

In Eq. (8) we use the relation [6]

$$h^I - h^{II} = C_{p\,\mathrm{ef}} (T^I - T^{II}).$$

Equation (8) has one real root

$$m = A_1 + A_2, \quad A_{1,2} = \sqrt[3]{-e/2 \pm \sqrt{(d/3)^3 + (e/2)^2}}. \tag{9}$$

Thus, from Eqs. (8) and (9) with the unknowns T^I, T^{II}, and p we determine the velocities

$$V^{\mathrm{I}} = m/\rho^{\mathrm{I}} = \frac{mR_g T^{\mathrm{I}}}{p}, \qquad V^{\mathrm{II}} = m/\rho^{\mathrm{II}} = \frac{mR_g T^{\mathrm{II}}}{p},$$
$$T^{\mathrm{I}} = T_w, \quad T^{\mathrm{II}} = bT_w, \quad p = \rho_\infty V_\infty^2 (1 - \varepsilon/2). \tag{10}$$

The flow in sublayers I and II is described by the system of equations (3) with T_0 = const and I_0 = const and with boundary conditions for sublayer I

$$\zeta^{\mathrm{I}} = 0 : u_1^{\mathrm{I}}(0) = 0, \;\; \rho_0^{\mathrm{I}} f_0^{\mathrm{I}} = W = \rho_w V_w / \rho_\infty V_\infty;$$
$$\zeta^{\mathrm{I}} = 1 : f_0^{\mathrm{I}}(1) = V_1 = V^{\mathrm{I}} / V_\infty \omega^2 \tag{11}$$

and for sublayer II

$$\zeta^{\mathrm{II}} = 0 : u_1^{\mathrm{II}}(0) = u_1^{\mathrm{I}}(1), \; f_0^{\mathrm{II}}(0) = V_2 = V^{\mathrm{II}} / V_\infty \omega^2; \; \zeta^{\mathrm{II}} = 1 : f_0^{\mathrm{II}}(1) = 0. \tag{12}$$

The solution is described by Eq. (5), and here the boundary conditions (11) and (12) give:

a) for sublayer I

$$c_1^{\mathrm{I}} = -\left(\frac{2\delta}{\omega}\right)^2 \frac{2\beta}{W}, \quad c_2^{\mathrm{I}} = 0,$$
$$\delta^{\mathrm{I}} = \frac{\omega}{A^{\mathrm{I}}} \sqrt{(f_0^{\mathrm{I}}(0) - V_1) f_0^{\mathrm{I}}(0)}, \;\; A^{\mathrm{I}} = \left(\frac{2\beta}{\rho_0^{\mathrm{I}}}\right)^{1/2}; \tag{13}$$

b) for sublayer II

$$c_1^{\mathrm{II}} = \frac{2}{c_2^{\mathrm{II}} - 1} \frac{2\delta^{\mathrm{II}}}{\omega} A^{\mathrm{II}}, \quad c_2^{\mathrm{II}} = \frac{u_1^{\mathrm{I}}(1)}{u_1^{\mathrm{I}}(1) - A^{\mathrm{II}}},$$
$$\delta^{\mathrm{II}} = \frac{\omega V_2}{A^{\mathrm{II}} + u_1^{\mathrm{I}}(1)}, \qquad A^{\mathrm{II}} = \left(\frac{2\beta}{\rho_0^{\mathrm{II}}}\right)^{1/2}. \tag{14}$$

Thus, having $|q_\infty| = q^*$, T_w, ρ_∞, V_∞, $\rho_w V_w$, R_c and using Eqs. (5), (8), (9), (13), and (14), we can obtain an approximate analytical solution of the problem.

The results of numerical solution and approximate theory are compared in Fig. 2 ($\delta(q^*)$) and Fig. (3) (u_1, f_0).

The solid curve of Fig. 4 shows the position of the discontinuity, obtained approximately.

It is clear that the less is δ_{abs}, the better will the approximate results agree with the exact. Nevertheless, for $\delta_{abs} \lesssim 0.2$ the results of the numerical and the approximate calculations agree not only qualitatively, but also quantitatively (in position of the absorption layer the discrepancy is less than 15%, and in displacement thickness it is less than 30%).

The above approximate method is based on the premise that the blown layer consists of two sublayers with T_0 = const, p = const, I = const and that the quantity δ_{abs} the absorption zone thickness, is small. After obtaining the approximate solution one should check that the assumptions made correspond to the solution obtained.

To do this, starting from the approximate solution, one must calculate δ_{abs}. If δ_{abs} is small and the absorption front lies within the blown layer, the solution can be considered acceptable.

To determine δ_{abs} we assume that between $T_0 = 1$ and $T_0 = b$ the temperature varies linearly (see Fig. 1). Let the absorption zone coincide with the intercept $[\zeta_1, \zeta_2]$. Then from the radiative transfer equation and the linearity of T_0 it follows that:

$$I_0 = c \exp [\tau_0 \int k(p, T_0) d\zeta], \quad T_0 = \frac{b-1}{\zeta_2 - \zeta_1}(\zeta - \zeta_1) + 1.$$

Taking into account that $I_0(\zeta_2) = 1$, we have:

$$I_0 = \exp\left\{\tau_0 \frac{p(b-1)}{\zeta_2-\zeta_1}\left[\zeta\left(\zeta_2-\frac{\zeta^2}{2}\right)-\frac{\zeta_2^2}{2}\right]\right\},$$

$$I_0(\zeta_1) = \exp\left\{-\frac{\tau_0 p(b-1)}{2}\delta_{abs}\right\},$$

$$\delta_{abs} = \zeta_2 - \zeta_1.$$

From the condition $I_0(\zeta_1) \ll 1$ we determine δ_{abs}. For example, let $I_0(\zeta_1) = 10^{-n}$, then

$$\delta_{abs} = \frac{n}{\tau_0}\frac{2\ln 10}{p(b-1)} = \frac{n}{\delta k^* R_c}\frac{2\ln 10}{p(b-1)},$$

where δ is the displacement thickness. For example, a comparison of $\delta_{abs,num}$, the thickness of the absorption layer calculated numerically, and $\delta_{abs,app}$ obtained by the above method (for the series of parameters see Fig. 3) gives (n = 2):

q^*, W/m^2	$\delta_{abs, num}$	$\delta_{abs, app}$
$2{,}385\cdot10^7$	0,12	0,166
$7{,}155\cdot10^7$	0,16	0,176
$1{,}908\cdot10^8$	0,17	0,193

The satisfactory agreement of $\delta_{abs,num}$ and $\delta_{abs,app}$ shows the validity of the approximate method for the region of variation of the parameters examined. We note that the approximate method described above can also be applied in the case of a different dependence of the absorption coefficient on pressure and temperature, and it is necessary only that k(p, T) be a monotonically decreasing function of temperature, that the radiative absorption occur in a narrow band and that the absorption cease after the gas reaches the critical temperature T_{cr}.

NOTATION

λ, wavelength of the external radiation; ρ, v, p, h, T, density, velocity, pressure, enthalpy and temperature of the gas; x, y, coordinates respectively along the body surface and normal to it; $\omega = V_w^*/V_\infty\cdot(\rho_w^*/\rho_\infty)^{1/2}$, $\varepsilon = \rho_\infty/\rho_s$, small parameters; R_c, radius of body curvature at the stagnation point; r, distance from the axis of symmetry; $\nu = 1, 2$, corresponds to planar and axisymmetric flow; q, radiant energy flux density; I, radiative intensity; α, angle between the axis of symmetry and the tangent to the body; R_g, gas constant; k, absorption coefficient; δR_c, thickness of the blown layer on the axis of symmetry; τ_0, optical thickness; c_p, specific heat at constant pressure. Subscripts: ∞, *, s, and w, parameters respectively in the incident flow, characteristic, behind the shock wave, and at the body.

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